

In complete sentences, using proper English and mathematical notation,

SCORE: _____ / 4 PTS

state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

IF f IS CONTINUOUS ON $[a, b]$

① IF $g(x) = \int_a^x f(t) dt$, THEN $g'(x) = f(x)$ ON (a, b)

② IF $F'(x) = f(x)$, THEN $\int_a^b f(x) dx = F(b) - F(a)$

IF F' IS CONTINUOUS ON $[a, b]$, THEN $\int_a^b F'(x) dx = F(b) - F(a)$

Answer the following questions about the definition of the definite integral as presented in lecture.

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(Your answers may refer to the fact that the definite integral equals the area under a curve which is above the x -axis.)

[a] Why is there a summation in the definition ?

THE AREA IS APPROXIMATED BY THE SUM OF AREAS OF
RECTANGLES

[b] What does the inequality regarding x_i^* ($a + (i-1)\Delta x \leq x_i^* \leq a + i\Delta x$) mean ? Recall that i goes from 1 to n .

ONE x -VALUE IS TAKEN FROM EACH SUBINTERVAL
TO CREATE A RECTANGLE (WITH HEIGHT EQUAL
TO THE CORRESPONDING f VALUE)

Let $g(x) = \int_1^x f(t) dt$, where f is the function whose graph is shown on the right.

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- [a] Find $g'(-1)$. Explain your answer very briefly.

① POINT EACH

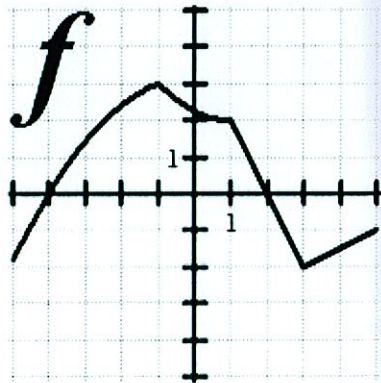
$$\underline{g'(-1) = f(-1) = 3}$$

- [b] Find all intervals over which g is concave down. Explain your answer very briefly.

$$\underline{g'(x) = f(x) \text{ IS DECREASING ON } (-1, 3)}$$

- [c] Find the x -coordinates of all local minima of g . Explain your answer very briefly.

$$\underline{g'(x) = f(x) \text{ CHANGES FROM NEGATIVE TO POSITIVE}} \\ \text{AT } \underline{x = -4}$$



A tank is being used to store gas. The pressure inside the tank changes as the gas changes temperature.

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When the temperature of the gas is t° Celsius, the pressure increases by $p(t)$ Pascals per degree Celsius that the temperature rises.

What is the meaning of the equation $\int_{55}^{70} p(t) dt = 40$ in this situation ?

NOTES: Your answer must use all three numbers from the equation, along with correct units.

Your answer should NOT use “ t ”, “ $p(t)$ ”, “integral”, “antiderivative”, “rate of change” or “derivative”.

THE PRESSURE IN THE TANK INCREASES 40 PASCALS
IF THE TEMPERATURE OF THE GAS RISES FROM 55°C TO 70°C

If $k(x) = \int_{-2x}^{x^4} \sqrt{1+t^3} dt$, find $k'(-1)$.

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$$k'(x) = (x^4)' \sqrt{1+(x^4)^3} - (-2x)' \sqrt{1+(-2x)^3}$$

$$= \underbrace{4x^3}_{\textcircled{\frac{1}{2}}} \underbrace{\sqrt{1+x^{12}}}_{\textcircled{1}} + \underbrace{2}_{\textcircled{\frac{1}{2}}} \underbrace{\sqrt{1-8x^3}}_{\textcircled{1}}$$

$$k'(-1) = -4\sqrt{2} + 2\sqrt{9}$$

$$= \underbrace{6-4\sqrt{2}}_{\textcircled{1}}$$

Evaluate the following integrals.

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SUBTOTAL
= 4 POINTS

[a] $\int \frac{(3-x)^2}{\sqrt{x}} dx$

$$= \int \frac{9-6x+x^2}{x^{\frac{1}{2}}} dx$$

$$\textcircled{1} \int (9x^{-\frac{1}{2}} - 6x^{\frac{1}{2}} + x^{\frac{3}{2}}) dx$$

$$= 9 \cdot 2x^{\frac{1}{2}} - 6 \cdot \frac{2}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$$

$$\textcircled{1} 18x^{\frac{1}{2}} - \textcircled{1} 4x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$$

SUBTOTAL
= 2 POINTS

[c] $\int_{-\pi}^{\pi} \sin^3 x \cos^5 x dx$

$$\textcircled{1} \boxed{\sin^3(-x) \cos^5(-x) = (-\sin x)^3 \cos^5 x}$$

$$= -\sin^3 x \cos^5 x$$

INTEGRAND IS ODD + CONTINUOUS

$$\text{INTEGRAL} = \textcircled{0} \textcircled{\frac{1}{2}}$$

SUBTOTAL
= 4 POINTS

[b] $\int_{-1}^1 \frac{5x^2}{6-2x^3} dx$

NOT ODD

$$\textcircled{\frac{1}{2}} \boxed{u = 6 - 2x^3} \begin{cases} x=1 \rightarrow u=4 \\ x=-1 \rightarrow u=8 \end{cases}$$

$$\frac{du}{dx} = -6x^2$$

$$dx = \frac{-du}{6x^2}$$

$$\frac{5x^2}{6-2x^3} dx = \frac{5x^2}{6-2x^3} \cdot \frac{-du}{6x^2}$$

$$= -\frac{5}{6} \frac{1}{u} du$$

$$\int_8^4 -\frac{5}{6} \frac{1}{u} du = -\frac{5}{6} \ln|u| \Big|_8^4$$

$$= -\frac{5}{6} (\ln 4 - \ln 8) \textcircled{1}$$

$$= -\frac{5}{6} \ln \frac{1}{2} = \frac{5}{6} \ln 2$$

EITHER ONE OK $\textcircled{\frac{1}{2}}$